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Determining Strictly Positive Realness from System Modal Characteristics

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I. Introduction

THE direct model reference adaptive control (DMRAC) and direct adaptive disturbance rejection (DADR) theorems for both continuous and discrete time rely on the existence of an output feedback gain that makes the closed-loop system to be strictly positive real (SPR) [1]. As with many existence proofs it is not generally necessary to know the exact value of the output feedback gain which causes the closed-loop system to be SPR. However, it is clear only certain systems can be made SPR with the use of output feedback. For example, if a system is a nonminimum phase, the system cannot be made SPR with output feedback because the use of output feedback does not change the location of the transmission zeros of the system [2]. Therefore, the question arises, *what types of systems can be made SPR with output feedback?* The first theorem in this Note shows that all single mode, minimum phase, single-input/single-output (SISO) systems which are controllable and observable can be made SPR with output feedback. Furthermore, a bound on the necessary gain is derived in the theorem.

Because stability is a necessary, but not sufficient, condition for SPR, the output feedback gain must always stabilize the system if it is unstable. Notice arbitrary pole placement is not required, that is, in cases where the Kimura–Davison [3] theorem does not apply, a system may be made SPR without arbitrary pole placement.

There are a number of papers on the subject describing when a linear system is SPR. For example, see Wen [4], or Tao and Ioannou [5]. This paper extends the work done by Livneh and Slater [6]. Livneh and Slater demonstrate how linear systems with N states and collocated actuators and sensors are SPR when a certain inequality is satisfied for each mode. The inequalities of Livneh and Slater relate the damping of each system mode to the ratio of the velocity output gain with the position output gain of the system. If the system is SPR, the velocity/position output gain ratio for each mode must be positive and less than the modal damping for each mode of the system. Livneh and Slater assume the linear system has only force inputs to the system. In other words, the input matrix for the linear system has a block of zeros multiplying the input function so that the velocity equation of the system is identically satisfied. In this Note the more general case when the system has some velocity input is considered.

First we refer to the standard state-space system,

$$\dot{x} = A\underline{x} + B\underline{u}, \qquad \underline{y} = C\underline{x} + D\underline{u} \tag{1}$$

Equation (1) will also be referred to as the quadruple (A, B, C, D) for shorthand. For a system with output feedback the control law,

$$\underline{u} = K\underline{y} \tag{2}$$

is assumed.

II. Strict Positive Realness for a Single Mode SISO System

Assume the system in Eq. (1) has a single mode, is single input/single output, and is stable, that is, the plant has a certain amount of inherent modal damping. It is trivial to verify a gain K exists such that the closed-loop state matrix $A - BKC$ is stable. In fact, if the gain K is zero, then the state matrix reduces to $A - BKC = A$ which is already known to be exponentially stable. However, as will be shown, a different choice of K can make the closed-loop state matrix $A - BKC$ stable and SPR if A is not already SPR.

The system in Eq. (1) is SPR if matrices P and Q exist which satisfy the reduced Lur'e, or Kalman–Yacubovic (K-Y) equations (when $D = 0$), given as

$$A^T P + PA = -Q^E Q - L, \qquad BP = C^T$$

Following the work of Wen [4], if the conditions,

$$G(j\omega) \equiv T(j\omega) + T^*(j\omega) > 0, \quad \text{and} \quad \lim_{\omega\to\infty} \omega^2 G(j\omega) > 0 \tag{3}$$

are satisfied, where

$$T(s) \equiv C(sI - A)^{-1}B + D$$

is the transfer function of the system in Eq. (3), then the K-Y conditions are satisfied.

Unfortunately, finding an output feedback gain that makes the system in Eq. (1) stable and SPR is difficult in all but the simplest case. However, if we consider a system with only one mode, we can find conditions on the input and output matrices, as well as the output feedback gain K, such that the plant with output feedback is SPR and stable. Considering the single mode plant is useful, because it will give us some insight into physical systems that are SPR.

Consider the open loop system,

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -d \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \Phi \end{bmatrix}, \qquad C = [\alpha\Phi \;\; \Phi], \qquad D = 0 \tag{4}$$

where ω_n is the frequency of the mode being considered and d is the damping in the system. In this case the sensors and actuators are collocated because the actuator and sensor influence gains are

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equivalent. For our purposes, we will assume the system has modal damping and as such,

$$d = 2\zeta\omega_n$$

The system in Eq. (4) is in modal form and it is forced by a single input through the input gain Φ. The output of the system is a linear combination of displacement and velocity through the output gains $\alpha\Phi$ and Φ, respectively.

Theorem 1: The system in Eq. (4), with output feedback given by Eq. (2) is stable and SPR if and only if, $\alpha > 0$, and

$$K > \max\left\{\frac{-\omega_n^2}{\Phi^2\alpha}, \frac{\alpha - d}{\Phi^2}\right\}$$

Proof:

First define,

$$A_c \equiv A - BKC$$

and

$$T(s) \equiv C(sI - A_c)^{-1}B + D \tag{5}$$

Substituting (A_c, B, C), from Eq. (4) into Eq. (5) produces

$$T(s) = \frac{\Phi^2(s + \alpha)}{s^2 + (d + K\Phi^2)s + (\omega_n^2 + K\Phi^2\alpha)} \tag{6}$$

Equation (6) represents a stable system if

$$d + K\Phi^2 > 0, \quad \text{and} \quad \omega_n^2 + K\Phi^2\alpha > 0 \tag{7}$$

In Eq. (7) the sign of α must be considered. If $\alpha > 0$, then

$$K > \frac{-d}{\Phi^2}, \quad \text{and} \quad K > \frac{-\omega_n^2}{\Phi^2\alpha}$$

Conversely if $\alpha < 0$, then

$$K > \frac{-d}{\Phi^2}, \quad \text{and} \quad K < \frac{-\omega_n^2}{\Phi^2\alpha} \tag{8}$$

Using Eq. (5) in the first equation of (3), we get

$$G(j\omega) = \frac{\Phi^2[(d + K\Phi^2 - \alpha)\omega^2 + \alpha(\omega_n^2 + K\Phi^2\alpha)]}{P(j\omega)P(-j\omega)} \tag{9}$$

where

$$P(s) \equiv s^2 + (d + K\Phi^2)s + (\omega_n^2 + K\Phi^2\alpha)$$

Notice the denominator of Eq. (9) is quadratic and positive for all ω, since

$$P(j\omega)P(-j\omega) = (\omega_n^2 - \omega^2 - K\Phi^2\alpha)^2 + (d + K\Phi^2)^2\omega^2 > 0 \quad \forall\ \omega \in \Re$$

Therefore, for $G(j\omega)$, positive definite requires

$$d + K\Phi^2 - \alpha > 0, \quad \text{and} \quad \alpha(\omega_n^2 + K\Phi^2\alpha) > 0$$

If $\alpha > 0$, then

$$K > \frac{\alpha - d}{\Phi^2}, \quad \text{and} \quad K > \frac{-\omega_n^2}{\Phi^2\alpha} \tag{10}$$

Also if $\alpha < 0$, then

$$K > \frac{\alpha - d}{\Phi^2}, \quad \text{and} \quad K > \frac{-\omega_n^2}{\Phi^2\alpha} \tag{11}$$

Equations (10) and (11) imply that no matter the sign of α,

$$K > \frac{-\omega_n^2}{\Phi^2\alpha}$$

Therefore, the final condition in Eq. (8) cannot satisfy the constraint, $G(j\omega) > 0$, and so α must be positive. Also, since $\alpha > 0$,

$$K > \frac{\alpha - d}{\Phi^2} > \frac{-d}{\Phi^2} \tag{12}$$

Also,

$$\lim_{\omega\to\infty} \omega^2 G(j\omega) = \lim_{\omega\to\infty} \omega^2 \frac{\Phi^2[(d + K\Phi^2 - \alpha)\omega^2 + \alpha(\omega_n^2 + K\Phi^2\alpha)]}{(\omega_n^2 - \omega^2 - K\Phi^2\alpha)^2 + (d + K\Phi^2)^2\omega^2} = \Phi^2(d + K\Phi^2 - \alpha) > 0$$

from Eq. (11). Therefore the system in Eq. (4) is stable and SPR, since the conditions in Eq. (3) are satisfied. This completes the proof. □

Theorem 1 demonstrates that the right choice of output feedback gain K stabilizes the system in Eq. (4) and satisfies the SPR condition. In the case when the system in Eq. (4) is stable and $K = 0$ the condition in Eq. (12) of Theorem 1 reduces to the condition given by Livneh and Slater [6], namely,

$$0 < \alpha < d$$

Therefore, providing output feedback for the system from Eq. (4) expands the range of the constant α. In other words, the system can have more position output and still remain SPR. However, the requirement of $\alpha > 0$ is the same as requiring that the system be minimum phase. Therefore, for the single mode case, the system must have a stable zero. How this result extends to a multi-input/multi-output (MIMO) system or a system with more then one mode is not clear and is presently considered.

III. Strict Positive Realness for an *N*th Order Square MIMO System

Consider an Nth order, MIMO system with $N_m = \frac{N}{2}$ modes which is stable without output feedback. In this case the system under consideration is not necessarily SPR. Define (A, B, C, D) for the system from Eq. (1) as

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & A_{Nm} \end{bmatrix}, \qquad B = \Gamma[b_1 \ \ b_2 \ \ \cdots \ \ b_p]$$
$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \end{bmatrix}, \qquad D = [0] \tag{13}$$

where the blocks A_i, b_i, and c_i, are defined as

$$A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -d_i \end{bmatrix}$$
$$b_i = [\beta_1\Phi_{i1} \ \ \Phi_{i1} \ \ \beta_2\Phi_{i2} \ \ \Phi_{i2} \ \ \cdots \ \ \beta_{N_m}\Phi_{iN_m} \ \ \Phi_{iN_m}]^T$$
$$c_i = [\alpha_1\Phi_{i1} \ \ \Phi_{i1} \ \ \alpha_2\Phi_{i2} \ \ \Phi_{i2} \ \ \cdots \ \ \alpha_{N_m}\Phi_{iN_m} \ \ \Phi_{iN_m}] \tag{14}$$

and ω_i and d_i are the modal frequency and damping for each mode. The number of blocks required is p and is the number of inputs and outputs of the system in Eq. (1). In Eq. (13) the constant Γ multiplying the input matrix is a matrix contained in the set $\Re^{N \cdot N}$. In Eq. (14) the constants Φ_{ik} are scalars. It is not necessary to assume modal damping, that is, $d_i = 2\varsigma\omega_i$. Also, there are currently no restrictions on α_i, β_i, and Γ. Because the system defined in Eqs. (13) and (14) must be stable but not necessarily SPR, we only require that $\omega_i > 0$ and $d_i > 0$.

Theorem 2: The system in Eq. (1) with (A, B, C, D) given by Eqs. (13) and (14) is SPR if and only if $\Gamma > 0$, and

$$d_k + \beta_k \omega_k^2 > \alpha_k > \frac{\beta_k \omega_k^2}{\beta_k d_k + 1} \quad \forall\ k = 1, 2, \ldots, N_m \tag{15}$$

Proof:

The transfer function matrix of Eq. (1) is given by

$$T(s) = C(sI - A)^{-1} B$$

Because $(sI - A)$ is block diagonal, the transfer function for each input to each output can be evaluated as

$$T_{ij}(s) = \sum_{k=1}^{N_m} \underline{\Phi}_{ik} \underline{\alpha}_k (sI - A_k)^{-1} \underline{\Gamma}_k \underline{\beta}_k \underline{\Phi}_{jk}^T$$

where

$$\underline{\Phi}_{ik} = [\Phi_{ik}\ \Phi_{ik}], \quad \underline{\alpha}_k = \begin{bmatrix} \alpha_k & 0 \\ 0 & 1 \end{bmatrix}, \quad \underline{\beta}_k = \begin{bmatrix} \beta_k & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \underline{\Gamma}_k = \Gamma_{kk} \mathbf{I}_2$$

where Γ_{kk} is a non-negative scalar. Now evaluate $(sI - A_k)^{-1}$ as

$$(sI - A_k)^{-1} = \frac{\begin{bmatrix} s + d_k & 1 \\ -\omega_k^2 & s \end{bmatrix}}{P_k(s)}$$

where

$$p_k(s) = s^2 + d_k s + \omega_k^2$$

Multiplying through by $\underline{\alpha}_k$ and $\underline{\beta}_k$ leaves

$$T_{ij}(s) = \sum_{k=1}^{N_m} \underline{\Phi}_{ik} \frac{\begin{bmatrix} \alpha_k \beta_k (s + d_k) & \alpha_k \\ -\beta_k \omega_k^2 & s \end{bmatrix}}{P_k(s)} \underline{\Gamma}_k \underline{\Phi}_{jk}^T$$

Finally, after some algebra, notice that

$$T_{ij}(s) = \sum_{k=1}^{N_m} \Phi_{ik} \frac{(\alpha_k \beta_k + 1)s + (\alpha_k \beta_k d_k + \alpha_k - \beta_k \omega_k^2)}{P_k(s)} \Gamma_{kk} \Phi_{jk} \tag{16}$$

The SPR condition given in Eq. (3) requires

$$G(j\omega) \equiv T(j\omega) + T^*(j\omega) > 0, \quad \text{and} \quad \lim_{\omega \to \infty} \omega^2 G(j\omega) > 0 \tag{17}$$

for all $\omega \in \Re$. Using Eq. (16) in the first condition of Eq. (17) results in

$$G_{ij}(j\omega) = \sum_{k=1}^{N_m} \Phi_{ik} \frac{(\beta_k \omega_k^2 + d_k - \alpha_k)\omega^2 + (\alpha_k \beta_k d_k \omega_k^2 + \alpha_k \omega_k^2 - \beta_k \omega_k^4)}{P_k(j\omega) P_k(-j\omega)} \Gamma_{k,k} \Phi_{jk}$$

Now let

$$X(j\omega) = \mathrm{diag}\left[\frac{(\beta_k \omega_k^2 + d_k - \alpha_k)\omega^2 + (\alpha_k \beta_k d_k \omega_k^2 + \alpha_k \omega_k^2 - \beta_k \omega_k^4)}{P_k(j\omega) P_k(-j\omega)}\right] \tag{18}$$

Therefore,

$$G(j\omega) = \Phi X(j\omega) \Gamma \Phi^T \tag{19}$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \cdots & \Phi_{1N_m} \\ \Phi_{21} & \Phi_{22} & & \vdots \\ \vdots & & \ddots & \\ \Phi_{p1} & \cdots & & \Phi_{pN_m} \end{bmatrix}$$

Because of the quadratic nature of Eq. (19), $G(j\omega) > 0 \quad \forall\ \omega \in \Re$, if $X(j\omega) > 0$ and $\Gamma > 0 \quad \forall\ \omega \in \Re$, that is, if $X(j\omega)$ and Γ are positive definite for all ω, then $G(j\omega) > 0$. $X(j\omega)$ is positive definite if each entry is positive definite, or if

$$\alpha_k \beta_k d_k \omega_k^2 + \alpha_k \omega_k^2 - \beta_k \omega_k^4 > 0, \quad \text{and} \quad (\beta_k \omega_k^2 + d_k - \alpha_k)\omega^2 > 0 \tag{20}$$

for all $\omega \in \Re$ and for all $k = 1, \ldots, N_m$, since the denominator in Eq. (18) is positive for all ω because each term in the denominator is squared. Equation (20) reduces to the two inequalities,

$$\alpha_k < d_k + \omega_k^2 \beta_k, \quad \text{and} \quad \alpha_k > \frac{\beta_k \omega_k^2}{\beta_k d_k + 1}$$

Therefore, α_k must satisfy the following inequalities for all $k = 1, 2, \ldots, N_m$:

$$d_k + \beta_k \omega_k^2 > \alpha_k > \frac{\beta_k \omega_k^2}{\beta_k d_k + 1} \tag{21}$$

The condition in Eq. (21) is satisfied by the assumption of Theorem 2. Finally,

$$\lim_{\omega \to \infty} \omega^2 G(j\omega) = \lim_{\omega \to \infty} \omega^2 \Phi X(\omega) \Gamma \Phi^T = \Phi(\lim_{\omega \to \infty} \omega^2 X(\omega)) \Gamma \Phi^T$$

and

$$\lim_{\omega \to \infty} \omega^2 X(\omega) = \lim_{\omega \to \infty} \omega^2 \mathrm{diag}\left[\frac{(\beta_k \omega_k^2 + d_k - \alpha_k)\omega^2 + (\alpha_k \beta_k d_k \omega_k^2 + \alpha_k \omega_k^2 - \beta_k \omega_k^4)}{P_k(j\omega) P_k(-j\omega)}\right]$$

Notice that

$$P_k(j\omega) P_k(-j\omega) = \omega^4 + (d_k - 2\omega_k^2)\omega^2 + \omega_k^4$$

Therefore,

$$\lim_{\omega \to \infty} \omega^2 X(\omega) = \lim_{\omega \to \infty} \mathrm{diag}\left[\frac{(\beta_k \omega_k^2 + d_k - \alpha_k)\omega^4 + (\alpha_k \beta_k d_k \omega_k^2 + \alpha_k \omega_k^2 - \beta_k \omega_k^4)\omega^2}{\omega^4 + (d_k - 2\omega_k^2)\omega^2 + \omega_k^4}\right]$$

Dividing both the numerator and the denominator by ω^4 results in

$$\lim_{\omega \to \infty} \mathrm{diag}\left[\frac{(\beta_k \omega_k^2 + d_k - \alpha_k) + (\alpha_k \beta_k d_k \omega_k^2 + \alpha_k \omega_k^2 - \beta_k \omega_k^4)\frac{1}{\omega^2}}{1 + (d_k - 2\omega_k^2)\frac{1}{\omega^2} + \frac{\omega_k^4}{\omega^4}}\right]$$

with limit,

$$\mathrm{diag}\,[\beta_k \omega_k^2 + d_k - \alpha_k] \tag{22}$$

The diagonal matrix in Eq. (22) is positive definite if each entry along the diagonal of the matrix is positive definite. But this condition is already satisfied by the condition in Eq. (21). Therefore, the system from Eq. (1) with (A, B, C, D) given by Eqs. (13) and (14) satisfies the conditions in Eq. (3) and is therefore SPR. This completes the proof. □

The system defined by the quadruple (A, B, C, D) as given by Eqs. (13) and (14) is a second order system with velocity actuator input because there is some velocity input into the system whose magnitude is given by $\beta \Phi_{ik}$ for all $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, N_m$. In other words, the system is not driven by simple force actuators. Theorem 2 does not assume the size of β_i for each mode. Each β_i term corresponds to a velocity input to each mode. The extent of the velocity leakage, and hence the magnitude of the β_i term, depends of the nature of the actuators used to control the structure and their exact placement. If the actuator used to control the structural system is acting in the linear range, that is, the actuator applies a nearly pure force, it is reasonable to expect each β_i is small compared to each Φ_{ik}. Regardless of the size of each β_i term compared to the Φ_{ik} terms Theorem 2 still holds. Theorem 2 can be used to test whether a system is SPR by straightforward calculation if the system modes, damping, and input and output matrices are known, and the input and output matrices have the required form. Testing the condition in Eq. (15) is computationally easy for large scale systems.

Finally, Theorem 2 does require the system be cast in a very specific form. The adjacent doubles of the B and C matrices in Eq. (14) only differ by a constant, that is, Φ_{ik} and $\alpha\Phi_{ik}$; this is not much of a constraint. One could easily imagine factoring out the constants α_i, β_i, and Γ. However, requiring the B and C matrices to contain the factor Φ_{ik} and differ only by a constant can be problematic. For real physical systems this idealization is not always possible. The B and C matrices in Eq. (14) can be factored into the following form:

$$B = \Gamma \underline{\beta}^D \underline{\Phi}, \qquad C = \underline{\Phi}^T \underline{\alpha}^D \tag{23}$$

where

$$\underline{\beta}^D = \mathrm{diag}(\underline{\beta}_k), \quad \underline{\alpha}^D = \mathrm{diag}(\underline{\alpha}_k), \quad \underline{\Phi} = \begin{bmatrix} \underline{\Phi}_{11} & \underline{\Phi}_{12} & \cdots & \underline{\Phi}_{1N_m} \\ \underline{\Phi}_{21} & \underline{\Phi}_{22} & \cdots & \underline{\Phi}_{2N_m} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\Phi}_{p1} & \underline{\Phi}_{p2} & \cdots & \underline{\Phi}_{pN_m} \end{bmatrix}$$

Given the SPR conditions via the Kalman–Yakubavich conditions, we require

$$PB = C^T$$

Using the results of Eq. (23), the K-Y conditions reduce to

$$P\Gamma \underline{\beta}^D \underline{\Phi} = (\underline{\Phi}\underline{\alpha}^D)^T$$

No extra insight into the requirements on $\underline{\beta}^D$ are gained here, but we see that the requirement on Γ being positive definite is not significant compared to the requirements on $\underline{\beta}^D$ and $\underline{\alpha}^D$ because the matrix P must also be strictly positive. More specifically, the term Γ provides an extra degree of freedom when it comes to casting the system in the form given by Eqs. (13) and (14).

IV. Conclusions

This Note presents two theorems which can be used to determine whether or not a system is SPR. Theorem 1 gives a very clear condition for when a system is SPR and stable with output feedback. In fact, Theorem 1 places a bound on how large the output feedback gain K must be so that the system with output feedback is SPR. Theorem 1 does rely on the system being minimum phase. However, Theorem 1 only applies to single mode systems, a serious limitation.

Theorem 2 provides SPR test conditions for any size system with velocity leakage in the input. Theorem 2 does not provide insight into the existence of an output feedback matrix which could make the system SPR if the system is minimum phase and not SPR. There are other methods of determining whether or not a system is SPR. For instance, solving the Riccati equation for a given system is sufficient to show that a system is SPR. However, when a system is very nearly SPR the Riccati equation is nearly singular and problems with numeric solution techniques arise. The method presented in Theorem 2 does not suffer from this defect. The method presented in this Note for determining if a system is SPR is one of many tools for use in linear systems analysis.

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